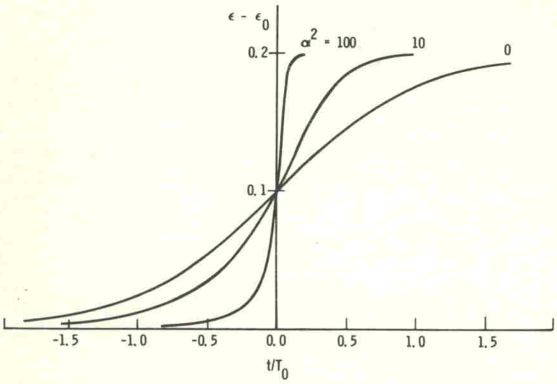

FIG. 7. Steady waveforms of amplitude $\epsilon_1-\epsilon_0=0.2$ in a material collapsing according to Eq. (24). The effect of increased strain-weighting of the collapse rate is illustrated. We have taken $\beta^2=10$.

shock formation. Values of $\mathscr{T}/T$ given by Eq. (21) are plotted as functions of wave amplitude for various values of β^2 on Fig. 5.

B. Strain-Weighted Collapse Rule

In a recent paper, Butcher[12] examined two steady-wave profiles propagated in a rigid polyurethane foam. His results showed that the rise time was a stronger function of wave amplitude than is implied by the linear collapse model, and he expressed this discrepancy in terms of the characteristic time, saying that it seemed to be shorter for the higher amplitude wave than for the one of lower amplitude. This tends to confirm our expectation that the characteristic time for collapse should be a decreasing function of strain. To explore the consequences of such an assumption, let us consider the collapse function

$$\phi=[\sigma-\sigma_E(\epsilon)][\sigma^* T(\epsilon)]^{-1} \quad , \qquad (22)$$

with $T(\epsilon)$ given by the simple relation

$$T(\epsilon)=T_0[1+\alpha^2(\epsilon-\epsilon_0)]^{-1} \qquad (23)$$

so that the collapse rate increases linearly with strain at any given overstress $\sigma-\sigma_E$.

The integral of Eq. (17) is readily evaluated for this collapse rule, and we obtain the implicit strain history

$$\frac{t}{T_0}=\frac{1}{\beta^2\epsilon_1}\left[\log_e\left(2\frac{\epsilon}{\epsilon_1}\right)-\frac{\epsilon_1\alpha^2}{1+\epsilon_1\alpha^2}\log_e\left(2\frac{1+\epsilon_1\alpha^2(\epsilon/\epsilon_1)}{2+\alpha^2\epsilon_1}\right)\right.$$
$$\left.-\frac{1}{1+\epsilon_1\alpha^2}\log_e\left(2\,(1-\epsilon/\epsilon_1)\right)\right] \quad , \qquad (24)$$

where we have taken $\epsilon_0=0$, $\sigma_0=0$, and $\sigma^*=\rho_0c_0^2$. As before, the steady wave speed is given by Eq. (18). The rise time $\mathscr{T}$, as defined in the previous example, is found to be

$$\frac{\mathscr{T}}{T_0}=\frac{1}{\beta^2\epsilon_1(1+\alpha^2\epsilon_1)}\left[5.889+\alpha^2\epsilon_1\log_e\left(19\,\frac{1+0.05\alpha^2\epsilon_1}{1+0.95\alpha^2\epsilon_1}\right)\right]. \qquad (25)$$

In Fig. 6 we have plotted strain histories of various amplitudes for $\alpha^2=100$ and $\beta^2=10$. To show the influence of variations of α^2 we have plotted strain histories in Fig. 7 for several choices of this parameter when ϵ_1 and β^2 are assigned the fixed values 0.2 and 10, respectively. Graphs of rise time as a function of wave amplitude are shown in Fig. 5 for $\alpha^2=0$ and are qualitatively the same but indicative of stronger quantitative dependence on amplitude as α^2 is increased. We note from Figs. 6 and 7 that the strengthened dependence of rise time on amplitude results primarily from a steepening of the upper portion of the wave profile.

C. Quadratic Collapse Rule

As an alternative to the introduction of strain-weighted collapse rules for increasing the dependence of rise time on amplitude we consider rules involving nonlinear dependence of the collapse rate on the overstress $\sigma-\sigma_E$. Specifically, we consider the case where

$$\phi_1=\frac{1}{\rho_0c_0^2T_1}[\sigma-\sigma_E(\epsilon)]\left(1+\frac{T_1/T_2}{\rho_0c_0^2}[\sigma-\sigma_E(\epsilon)]\right). \qquad (26)$$

It is clear that, for a given overstress, $\dot{\epsilon}$ will increase with decreasing T_2 (assuming $T_2>0$). When we use the function $\sigma_E(\epsilon)$ given by Eq. (11), calculation of the steady waveform is readily accomplished in the same way as before, and we arrive at the history

$$\frac{t}{T_1}=\frac{1}{\beta^2(\epsilon_1-\epsilon_0)}\log_e\left(\frac{\epsilon-\epsilon_0}{\epsilon_1-\epsilon}\right)$$
$$+\frac{1}{2r}\log_e\left(\frac{r-[\epsilon-\frac{1}{2}(\epsilon_1+\epsilon_0)]}{r+[\epsilon-\frac{1}{2}(\epsilon_1+\epsilon_0)]}\right), \qquad (27)$$

where

$$r=+\left[\frac{T_2}{T_1}\,\frac{1}{\beta^2}+\frac{1}{4}(\epsilon_1-\epsilon_0)^2\right]^{1/2} .$$

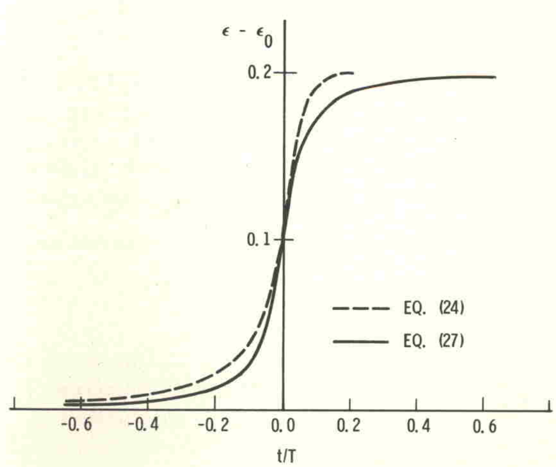

FIG. 8. Waveforms of the same amplitude and rise time calculated according to Eqs. (24) and (27). We have taken $\beta^2=10$, $T_0=T_1\equiv T$, $\alpha^2=100$, $T_2/T_1=0.00887$, and have $r/T=0.4459$.

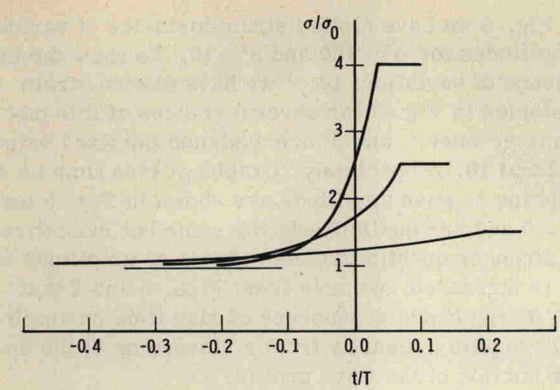

FIG. 9. Steady waveforms of various amplitudes in a locking material governed by the linear collapse rule.

Calculation of the rise time gives

$$\frac{\mathscr{T}}{T_1}=\frac{5.889}{\beta^2(\epsilon_1-\epsilon_0)}+\frac{1}{\beta^2 r}\log_e\frac{r-0.45\,(\epsilon_1-\epsilon_0)}{r+0.45\,(\epsilon_1-\epsilon_0)} \quad .$$

In the limit as $T_2\to\infty$ the waves governed by these equations become the same as those arising directly from the linear collapse rule. For finite values of T_2 the waveforms arising from the quadratic collapse theory show stronger dependence of rise time on amplitude while maintaining their symmetry.

As an indication of the sort of tailoring of waveforms that can be achieved by variations of the collapse rule, we have shown in Fig. 8 a waveform arising from the strain-weighted theory and one from the quadratic collapse theory. The equilibrium response curve and the basic time constants were chosen to be the same in each case and the parameters α^2 and T_2/T_1 were selected so that the rise time would be the same as well.

D. Locking Model

To get an indication of the influence of the equilibrium curve $\sigma_E(\epsilon)$ on steady-wave profiles, we consider wave propagation in a material governed by the strain-weighted collapse model of Eq. (22) and the locking equilibrium curve shown in Fig. 2. The locking model is of interest because it represents, in an exaggerated form, an aspect of the compaction behavior of many porous materials that is not well represented by the quadratic stress-strain curve.

The steady wave speed in a locking material is given by

$$\rho_0 V^2=(\sigma_1-\sigma_0)(\epsilon_s-\epsilon_0)^{-1} \quad .$$

Strain waves in a locking material all have the same amplitude, so we focus our attention on stress waves. Since $\sigma_E(\epsilon)=\sigma_0$ (for $\epsilon<\epsilon_s$), calculation of stress waveforms is particularly easy, and we obtain

$$t=\frac{\sigma^*(\epsilon_s-\epsilon_0)}{\sigma_1-\sigma_0}\int_{(\epsilon_s-\epsilon_0)/2}^{(\sigma-\sigma_0)(\epsilon_s-\epsilon_0)/(\sigma_1-\sigma_0)}\frac{T(\lambda)}{\lambda}\,d\lambda\,. \qquad (28)$$

The rise time, defined as before, is given by

$$\mathscr{T}=\frac{\sigma^*(\epsilon_s-\epsilon_0)}{\sigma_1-\sigma_0}\int_{0.05(\epsilon_s-\epsilon_0)}^{0.95(\epsilon_s-\epsilon_0)}\frac{T(\lambda)}{\lambda}\,d\lambda \quad .$$

In the particular case where $T(\epsilon-\epsilon_0)=T_0[1+\alpha^2\times(\epsilon-\epsilon_0)]^{-1}$, evaluation of the above integrals gives

$$t=\frac{T_0\sigma_0(\epsilon_s-\epsilon_0)}{\sigma_1-\sigma_0}\times\log_e\left(\frac{\sigma-\sigma_0}{\sigma_1-\sigma_0}\,\frac{2+\alpha^2(\epsilon_s-\epsilon_0)}{1+\alpha^2(\epsilon_s-\epsilon_0)\,(\sigma-\sigma_0)\,(\sigma_1-\sigma_0)^{-1}}\right) \qquad (29)$$

and

$$(\sigma_1-\sigma_0)\mathscr{T}=T_0\left[\sigma_0(\epsilon_s-\epsilon_0)\log_e\left(19\frac{1+0.05\alpha^2(\epsilon_s-\epsilon_0)}{1+0.95\alpha^2(\epsilon_s-\epsilon_0)}\right)\right]. \qquad (30)$$

In Fig. 9 we have shown some typical waveforms given by the special case of Eq. (29) in which $\alpha^2=0$, i.e., when collapse rule is linear, with the locking equilibrium curve. We see that these waveforms are very unsymmetrical, as one would expect from the fact that $\sigma-\sigma_E$ is largest at the peak amplitude of each wave. These waves also terminate abruptly at their peak amplitude because of the finite collapse rate there. Otherwise, their behavior is similar to that of the waves discussed previously. When we take $\alpha\neq 0$ the upper part of the waveform rises more steeply than in the case shown in Fig. 9.

V. EXPERIMENTAL DETERMINATION OF MATERIAL CONSTITUTION

In this section we discuss the inference of equilibrium stress-strain curves and collapse rules from experimental records of steady waveforms. For this purpose, we assume that one can experimentally establish the existence, form, and propagation velocity of such waves.

As a result of our discussion of this section it will become clear that, in each material, all steady waves associated with a given initial state (σ_0,ϵ_0) can be exactly reproduced by the present theory and that, for this reason, checks of the theory against experimentally determined steady waveforms give no information about its validity in the context of more general collapse processes.

A. Equilibrium Stress-Strain Curve

The equilibrium stress-strain curve is determined by applying Eq. (9) to the state behind each member of a family of steady waves of various amplitudes and measured velocities. If, for example, we have a particle-velocity history $u(t)$, u_1 is obtained as the limiting value of $u(t)$ as t becomes large. Equations (9) then give $\epsilon_1=\epsilon_0+[(u_1-u_0)/V]$ and $\sigma_1=\sigma_0+\rho_0(u_1-u_0)V$. This gives a point (σ_1,ϵ_1), and the locus of all such points, one obtained from each